Magnetic Hystersis of Ferromagnetic Materials

Ferromagnetics: Iron, nickel, cobalt → exhibit strong magnetic properties → used in permanent magnets.

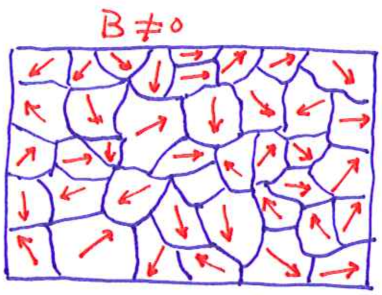
1) Their magnetic moment alignes along the external field.

2) They remain partially magnetized even after the field is removed.

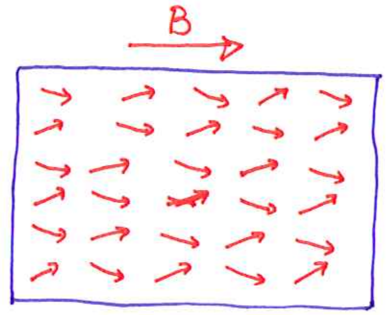
Magnetized domains: microscopic region ($\sim 10^{-10}\,m^3$) in the material where the magnetic moments are aligned in parallel ($\sim 10^{19}$ atoms)

In the absent of the magnetic field, the domains take random orientations and the total magnetic field becomes zero.

Domain walls: The boundary between adjacent domains consist of thin transition regions called domain wall.



Unmagnetized domains

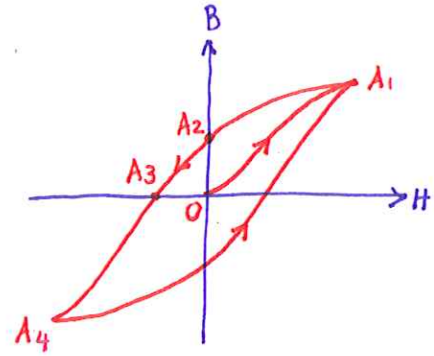


Magnetized domains

If the external magnetic field intensity is $\vec{H}$, and $\vec{B}$ is the total magnetic flux density in the material, $\vec{B}$ consists of two parts: $\mu_0 \vec{H}$ and $\mu_0 \vec{M}$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

If we apply the exteral field to an unmagnetized material and measure B, we will see such a curve:



O: unmagnetized state

As $\vec{H}$ is increased, $\vec{B}$ increases from point O to point A_1.

At this point almost all domains become aligned with $\vec{H}$.

A_1: saturation point

As $\vec{H}$ decreases to zero, $\vec{B}$ decreases but has some finite value at $\vec{H}=0$ as show with point A_2.

This value, at point A_2, is called residual flux density B_r. The iron material is now a permanent magnet. Reversing $\vec{H}$ and increasing in opposite direction, reduces $\vec{B}$ to zero at A_3.

If $\vec{H}$ is increased further, $\vec{B}$ is saturated at A_4 where the magnetic dipoles are almost all aligned in $\vec{H}$ direction. If $\vec{H}$ is now decreased to zero and is increased in opposite diretion, $\vec{B}$ goes from A_4 to A_1. This process is called magnetic hystersis.

hystersis: "to lag behind"

The hystersis loop indicates that $\vec{B}$ in ferromagnetic material is not only a function of $\vec{H}$ and it also depends on the history of the material.

Hard ferromagnetic materials: materials with wide hystersis loops. They are used in making permanent magnets for motors and generators.

Soft ferromagnetic materials: have narrow hystersis loops. They can easily be magnetized and demagnetized.

To demagetize a ferromagnetic material, it is subjected to several hystersis cycles while gradually decreasing the peak-to-peak range of the applied field.

Magnetic Boundary Conditions

we already learned that how D and E are at boundary of two dissimilar materials:

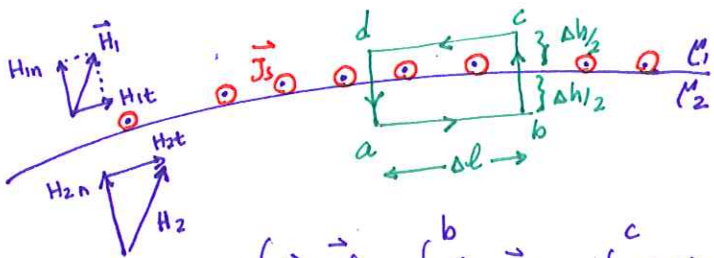
$$\oint_S \vec{D}\cdot d\vec{s} = Q \rightarrow D_{1n} - D_{2n} = \rho_s$$ we can write similarly for B:

$$\oint_S \vec{B}\cdot d\vec{s} = 0 \rightarrow \boxed{B_{1n} = B_{2n}}$$ → the normal component of $\vec{B}$ is continous.

$$\rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$$

For the tanjential part we have:

$$\oint_C \vec{H}\cdot d\vec{l} = \vec{J}$$ If we apply it across a boundary:



$$\oint_{\Delta h \to 0} \vec{H}\cdot d\vec{l} = \int_a^b \vec{H}_2\cdot d\vec{l} + \underbrace{\int_b^c \vec{H}\cdot d\vec{l}}_{\substack{\to 0 \\ \Delta h \to 0}} + \int_c^d \vec{H}_1\cdot d\vec{l} + \underbrace{\int_d^a \vec{H}\cdot d\vec{l}}_{\substack{\to 0 \\ \Delta h \to 0}} = I$$

$$= H_{2t}\Delta l - H_{1t}\Delta l = J_s \Delta l$$

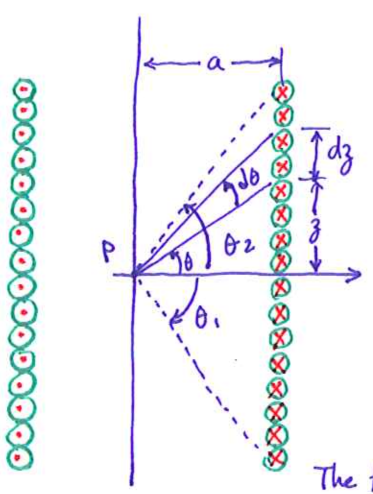
$$\Rightarrow \quad H_{2t} - H_{1t} = J_s \quad (A/m)$$

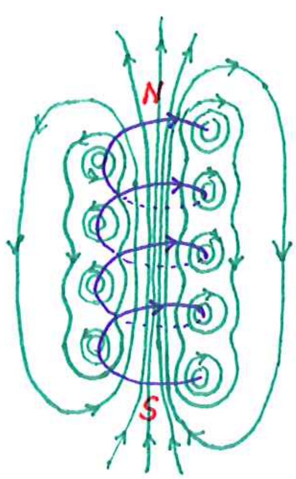
or: $$\boxed{\hat{n}_2 \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s}$$

If there is no current at interface: $H_{1t} = H_{2t}$.

Inductance

A typical inductor consists of a coil made from multiple turns of wire, which is called a Solenoid.





Magnetic Field in a Solenoid:

If we take each turn a circle, for a circular current we had:

$$\vec{H} = \hat{z}\,\frac{I' a^2}{2(a^2+z^2)^{3/2}}$$

n: # of turns per unit length

$$I' = I n\, dz \rightarrow d\vec{H} = \hat{z}\,\frac{n I a^2}{2(a^2+z^2)^{3/2}}\,dz$$

The total $\vec{B}$ at poin P is therefore obtained by integrating over z:

$$\left.\begin{aligned} z &= a\tan\theta \\ a^2+z^2 &= a^2 + a^2\tan^2\theta = a^2\sec^2\theta \\ dz &= a\sec^2\theta\, d\theta \end{aligned}\right\} \quad \vec{B} = \hat{z}\,\frac{\mu n I a^2}{2}\int_{\theta_1}^{\theta_2} d\theta\, \frac{a\sec^2\theta}{a^3\sec^3\theta} = \hat{z}\,\frac{\mu n I}{2}(\sin\theta_2 - \sin\theta_1)$$

If $l \gg a$: $\theta_1 \simeq -90^\circ$ and $\theta_2 \simeq 90^\circ \Rightarrow$

$$\boxed{\vec{B} \simeq \hat{z}\,\mu n I = \hat{z}\,\frac{\mu N I}{l} \qquad l \gg a \text{ long selonoid}}$$

$N = nl$ is the total number of turns over the length l.

Self-inductance: magnetic flux linkage of a coil or circuit with itself.

Mutual inductance: magnetic flux linkage in a circuit due to the magnetic fied generated by a current in another circuit.

usually the term "inductance" refers to self-inductance.

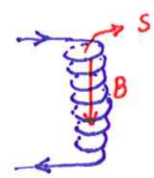
Self-Inductance

The magnetic flux Φ linking a surface S is: $\Phi = \int_S \vec{B} \cdot \vec{ds}$ (wb)

In a solenoid with an approximately uniform magnetic field, the flux linking a single loop is:

$$\Phi = \int_S \vec{B} \cdot \vec{ds} = \int_S \hat{z}\left(\mu \frac{NI}{l}\right) \cdot \hat{z}\, ds = \mu \frac{NI}{l} s$$

s is the cross section area of the loop



Magnetic flux linkage, Λ

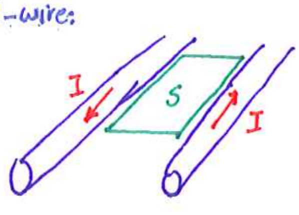
Λ is defined as the total magnetic flux linking a given circuit.

For a solonoid of N turns, the linking flux is for all loops:

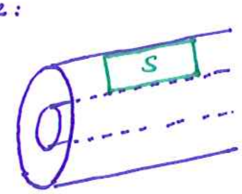
$$\Lambda = N\Phi = \mu N \overbrace{\frac{NIS}{l}}^{\Phi} = \mu \frac{N^2}{l} I S$$

If the structure consists of two separate conductors, the flux linkage Λ refers to the flux Φ through the closed surface between the two conductors:

Parallel-wire:



Coaxial line:



Self-inductance

for any conducting structure is the ratio of the magnetic flux linkage Λ to the current flowing through the structure:

$$\boxed{L \triangleq \frac{\Lambda}{I} \quad (H)}$$

For the solenoid, we have:

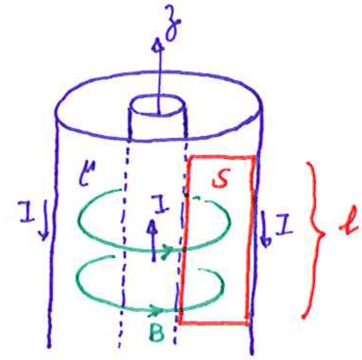
$$\boxed{L = \frac{\Lambda}{I} = \mu \frac{N^2}{l} S \quad (\text{solenoid})}$$

For two-conductor line, we have:

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{1}{I}\int_S \vec{B}\cdot d\vec{s}$$

Example

Find the inductance per unit length of a coaxial transmission line.



We calculated before that: $\vec{B} = \hat{\phi}\frac{\mu I}{2\pi r}$

$$\Phi = \int_S \vec{B}\cdot d\vec{s} = \int_a^b B\,(l\,dr) = l\int_a^b \frac{\mu I}{2\pi r}\,dr = \frac{\mu I l}{2\pi}\ln\left(\frac{b}{a}\right)$$

$\rightarrow L = \frac{\Lambda}{I}$ and for L per unit length: $L' = \frac{L}{l} = \frac{\Lambda}{lI} = \frac{\Phi}{lI}$

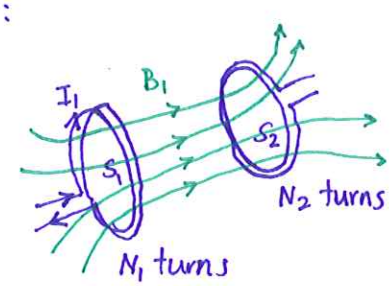
$$\rightarrow \boxed{L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)}$$

Mutual Inductance

Consider two closed loops with surfaces S_1 and S_2 and a current I_1 flowing through the first loop:

B_1 generated by I_1 results in a flux Φ_{12} through loop 2:

$$\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{s}$$

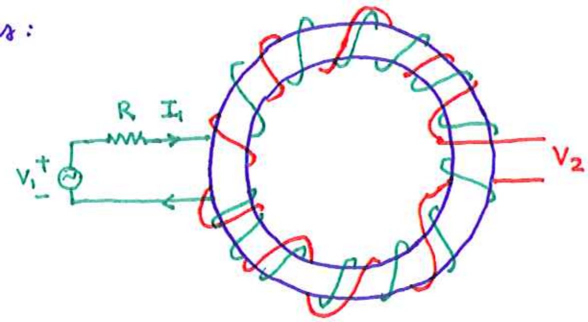


if loop 2 consists of N_2 turns all coupled by $\vec{B}_1$, the flux linkage through loop 2 due to $\vec{B}_1$ is:

$$\Lambda_{12} = N_2 \Phi_{12} = N_2 \int_{S_2} \vec{B}_1 \cdot \vec{ds}$$ So the mutual inductance associated with this magnetic coupling is given by:

$$\boxed{L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2}{I_1} \int_{S_2} \vec{B}_1 \cdot \vec{ds} \quad (H)}$$

Mutual inductance is important in transformers wherein the windings of two or more circuits share a common magnetic core, such as:



Magnetic Energy:

From the circuit we know that $v = L\frac{di}{dt}$. Power is vi, and the time integral of power is work:

$$W_m = \int p\,dt = \int iv\,dt = \int iL\frac{di}{dt}\,dt = L\int_0^I i\,di = \frac{1}{2}LI^2$$ magnetic energy stored in the inductor.

Let's consider a solenoid:

$$L = \mu \frac{N^2 S}{l}$$

$$B = \mu \frac{NI}{l} \rightarrow I = \frac{Bl}{\mu N}$$

$$W_m = \frac{1}{2}LI^2 = \frac{1}{2}\left(\mu\frac{N^2 S}{l}\right)\left(\frac{B^2 l^2}{\mu^2 N^2}\right) = \frac{1}{2}\frac{B^2}{\mu}(\overbrace{lS}^{v}) = \frac{1}{2}\mu H^2 v$$

So the magnetic energy density w_m is the magnetic energy W_m per unit volume:

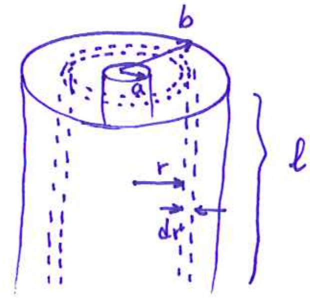
$$\boxed{w_m = \frac{W_m}{v} = \frac{1}{2}\mu H^2 \quad (J/m^3)}$$

This expression, eventhough derived for a solenoid, is valid for any medium v containing a material with permeability μ:

$$\boxed{W_m = \frac{1}{2}\int_V \mu H^2\,dv}$$

Example Magnetic Energy in a Coaxial Cable

for the coaxial cable we know: $H = \frac{B}{\mu} = \frac{I}{2\pi r}$



$$W_m = \frac{1}{2}\int_v \mu H^2 dv = \frac{1}{2}\int \mu \frac{I^2}{4\pi^2 r^2} dv = \frac{\mu I^2}{8\pi^2}\int_v \frac{dv}{r^2}$$

$$dv = 2\pi r \ell \, dr \rightarrow W_m = \frac{\mu I^2}{8\pi^2}\int_a^b \frac{1}{r^2} 2\pi r \ell \, dr$$

$$= \frac{\mu I^2 \ell}{4\pi} \ln\left(\frac{b}{a}\right)$$